The "Lucas Paradox" of Capital Flows

To put Lucas's point in a general setting, let us assume output depends on inputs of physical capital, human capital, and raw (efficiency-adjusted) labor:

$$Y = K^{\alpha} H^{\beta} \left(AL \right)^{1 - \alpha - \beta}$$

In the "intensive form" this becomes:

$$y = k^{\alpha} h^{\beta}.$$

We may therefore express the return to capital as its marginal product (assuming zero depreciation for simplicity),

$$r = \alpha k^{\alpha - 1} h^{\beta},$$

or since the production function implies that $k = y^{1/\alpha} h^{-\beta/\alpha}$,

$$r = \alpha \left(y^{1/\alpha} h^{-\beta/\alpha} \right)^{\alpha-1} h^{\beta}$$
$$= \alpha y^{\frac{\alpha-1}{\alpha}} h^{\frac{\beta}{\alpha}}.$$

For Lucas's first example, suppose that $\beta = 0$ and $A_{US} = A_{India}$. In 2012, US per capita income was roughly \$50,000 per year while India's was slightly under \$1,500 per year, for a ratio of 33.33 Thus, if $\alpha = 1/3$, we would conclude that the relative rate of return to capital in India is

$$\frac{r_{India}}{r_{US}} = \left(\frac{y_{US}}{y_{India}}\right)^{\frac{1-\alpha}{\alpha}} = (33.333)^2 \approx 1,111.$$

If so, capital would rush from the US to India, where huge investment would occur; but it does not.

Can human capital solve the problem? Again assuming that $A_{US} = A_{India}$, we would have

$$\frac{r_{India}}{r_{US}} = \left(\frac{y_{US}}{y_{India}}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{h_{India}}{h_{US}}\right)^{\frac{\beta}{\alpha}}.$$

Suppose for simplicity that $\beta = 0.33 = \alpha$. Then:

$$\frac{r_{India}}{r_{US}} = (1,111) \left(\frac{h_{India}}{h_{US}}\right).$$

Even if there is 100 times as much human capital in the US as in India, India's return on capital would still be more than ten times greater than that in the US., leaving a lot to be explained.

Finally, let's let the As differ. Then we would have to write

$$\frac{r_{India}}{r_{US}} = \left[\frac{(Y/L)_{US}}{(Y/L)_{India}}\right]^2 \left(\frac{A_{India}}{A_{US}}\right)^2 \left(\frac{h_{India}}{h_{US}}\right).$$

As an example, suppose the US has four times the human capital of India: $h_{India}/h_{US} = 1/4$. Then we can find the relative productivity level that equates marginal products of capital via

$$1 = \frac{1,111}{4} \left(\frac{A_{India}}{A_{US}}\right)^2,$$

or

$$\frac{A_{US}}{A_{India}} = \sqrt{\frac{1,111}{4}} = \sqrt{278} = 16.67.$$

If the US has ten times the human capital of India instead, this productivity ratio comes down to just over 10, which could be in a plausible range.

To gain perspective, we might look at the direct calculations of Hall and Jones.